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The Normal Distribution

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Topic Overview

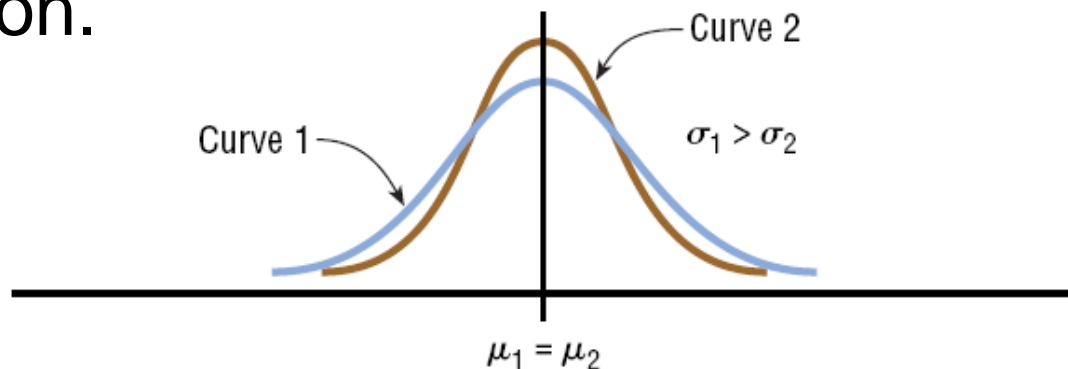
- Introduction
- Normal Distributions
- Applications of the Normal Distribution
- The Central Limit Theorem

Objectives

1. Identify the properties of a normal distribution.
2. Find the area under the standard normal distribution, given various z values.
3. Find probabilities for a normally distributed variable by transforming it into a standard normal variable.
4. Use the central limit theorem to solve problems involving sample means for large samples.

Normal Distributions

- The shape and position of the normal distribution curve depend on two parameters, the **mean** and the **standard deviation**.
- Each normally distributed variable has its own normal distribution curve, which depends on the values of the variable's mean and standard deviation.

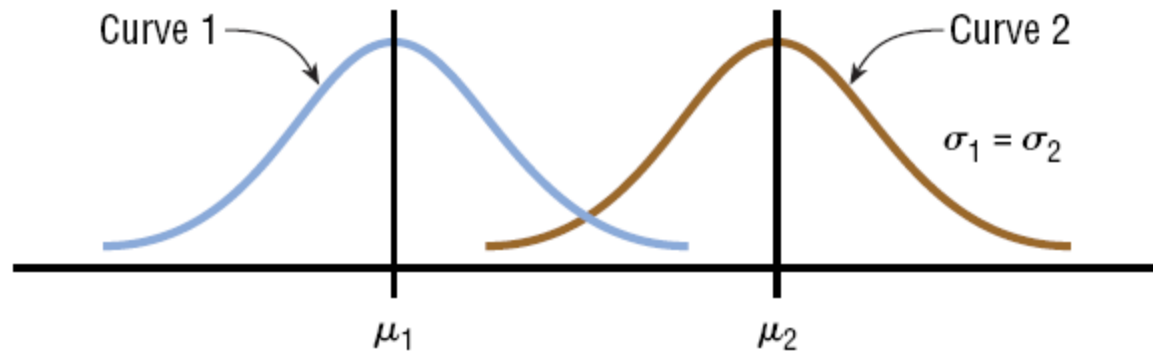


(a) Same means but different standard deviations

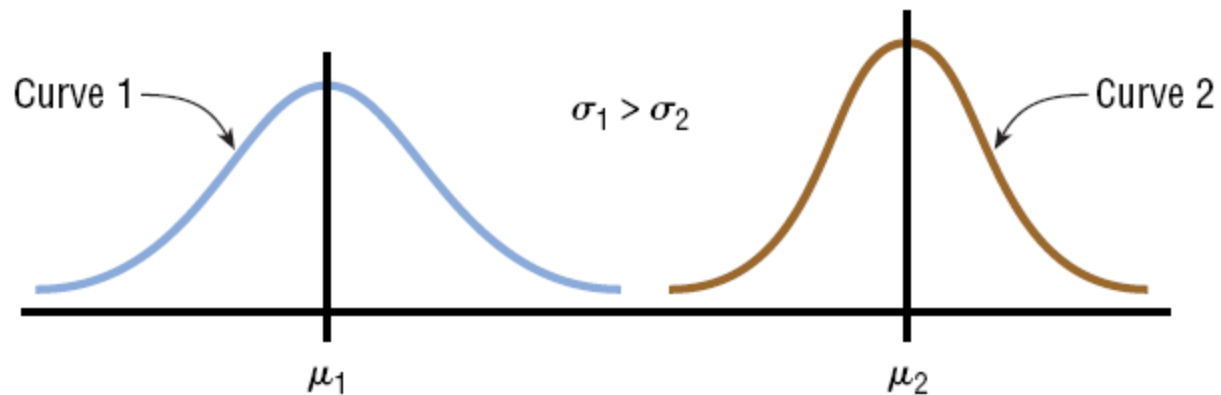
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Normal Distributions



(b) Different means but same standard deviations



(c) Different means and different standard deviations

Normal Distribution Properties

- The normal distribution curve is bell-shaped.
- The mean, median, and mode are equal and located at the center of the distribution.
- The normal distribution curve is **unimodal** (i.e., it has only one mode).
- The curve is symmetrical about the mean, which is equivalent to saying that its shape is the same on both sides of a vertical line passing through the center.

Normal Distribution Properties

- The curve is continuous—i.e., there are no gaps or holes. For each value of X , there is a corresponding value of Y .
- The curve never touches the x axis. Theoretically, no matter how far in either direction the curve extends, it never meets the x axis—but it gets increasingly closer.
- The total area under the normal distribution curve is equal to 1.00 or 100%.

Standard Normal Distribution

- Since each normally distributed variable has its own mean and standard deviation, the shape and location of these curves will vary. To simplify this, statisticians use the standard normal distribution.
- The **standard normal distribution** is a normal distribution with a mean of 0 and a standard deviation of 1.

z value (Standard Value)

The z value is the number of standard deviations that a particular X value is away from the mean. The formula for finding the z value is:

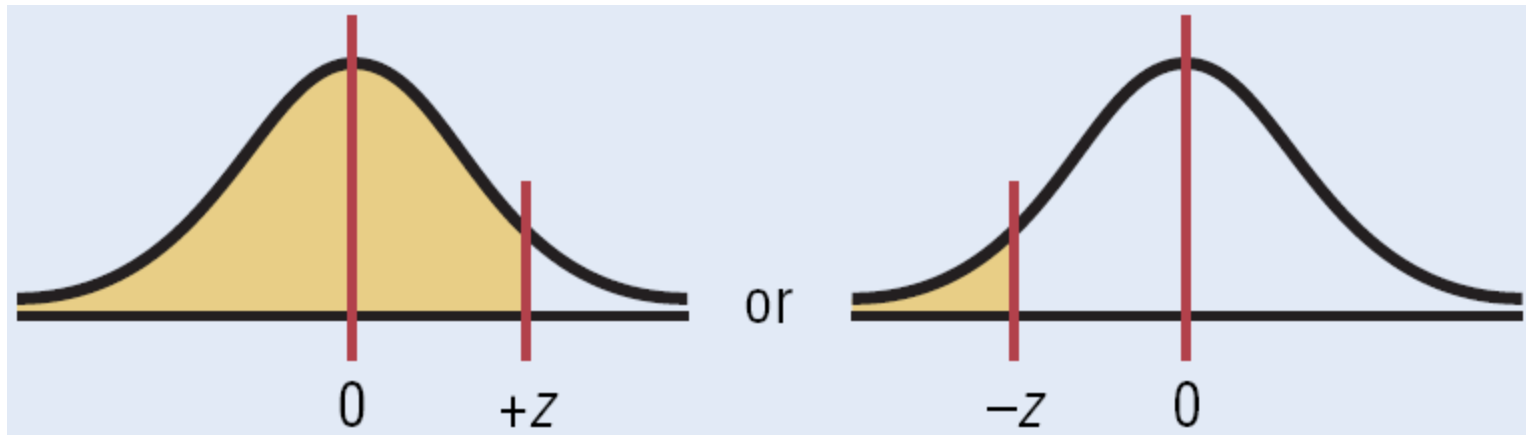
$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

$$z = \frac{X - \mu}{\sigma}$$

Area under the Standard Normal Distribution Curve

1. To the left of any z value:

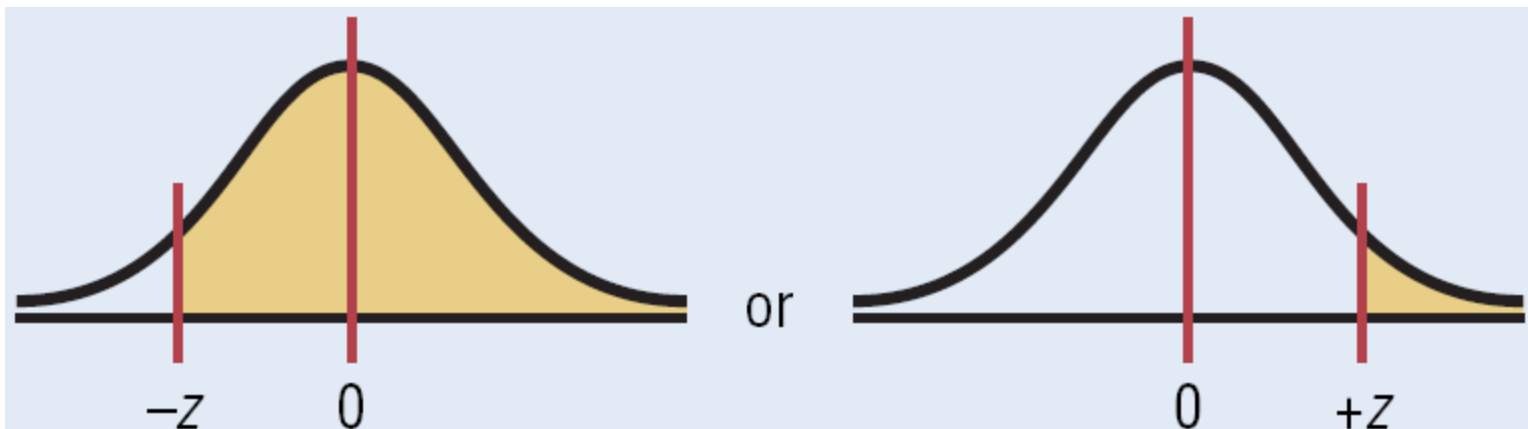
Look up the z value in the table and use the area given.



Area under the Standard Normal Distribution Curve

2. To the right of any z value:

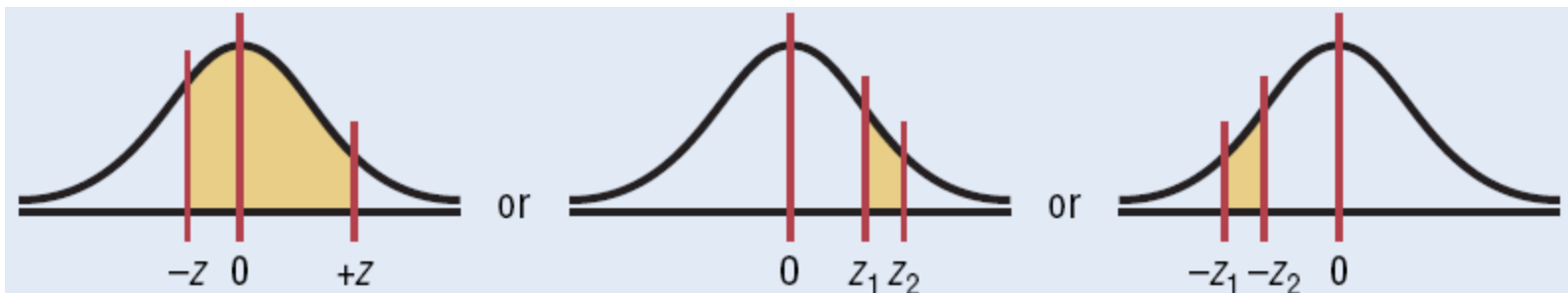
Look up the z value and subtract the area from 1.



Area under the Standard Normal Distribution Curve

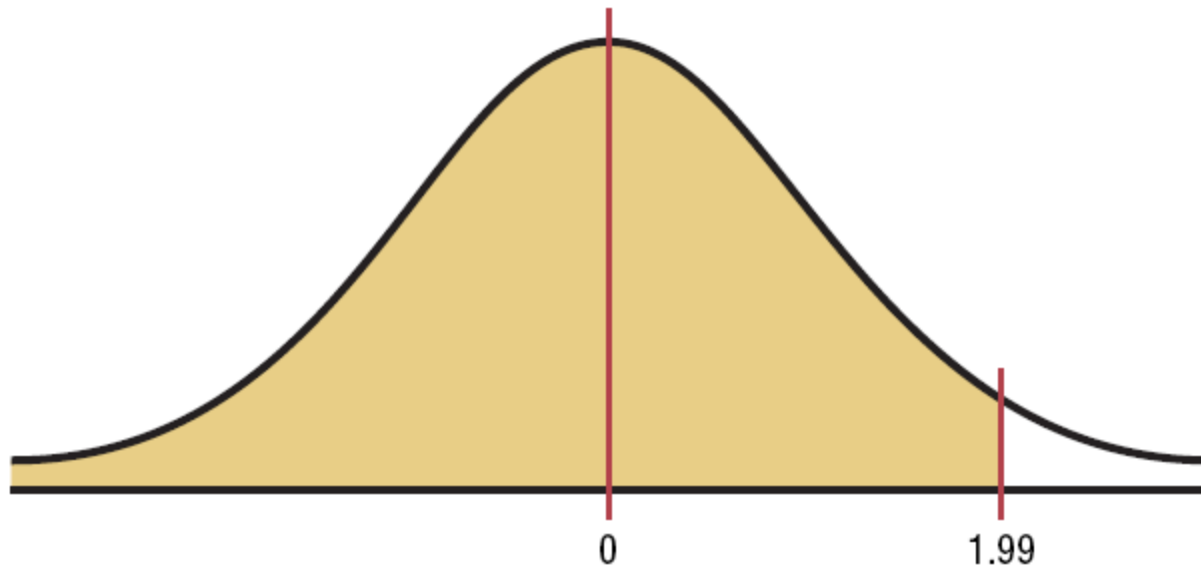
3. Between two z values:

Look up both z values and subtract the corresponding areas.



Area under the Curve

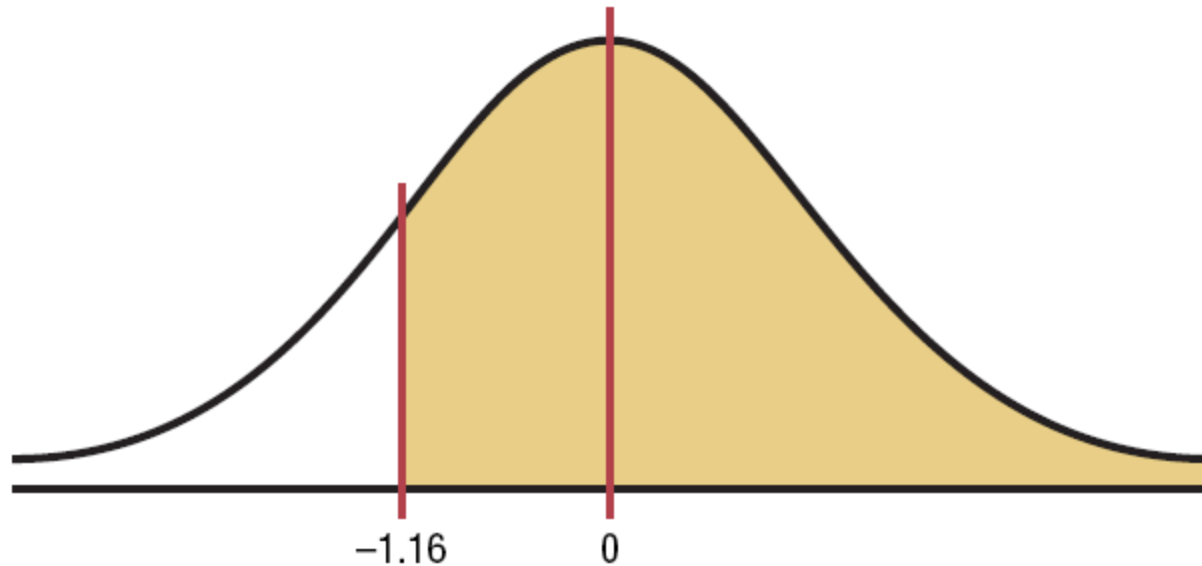
Find the area to the left of $z = 1.99$.



The area is .9767.

Area under the Curve

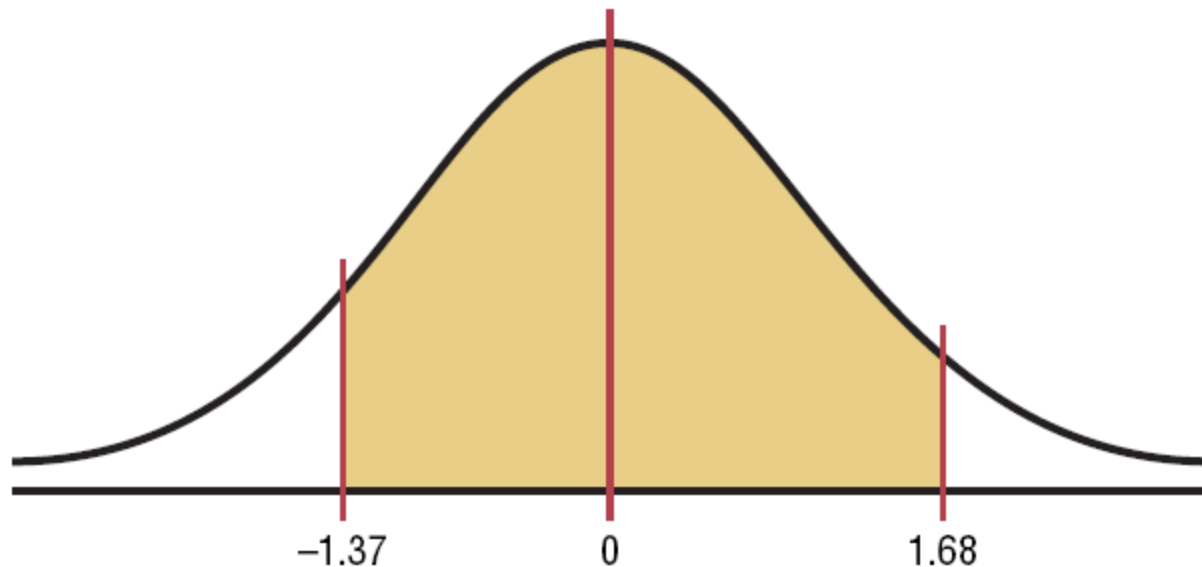
Find the area to right of $z = -1.16$.



The area is $1 - .1230 = .8770$.

Area under the Curve

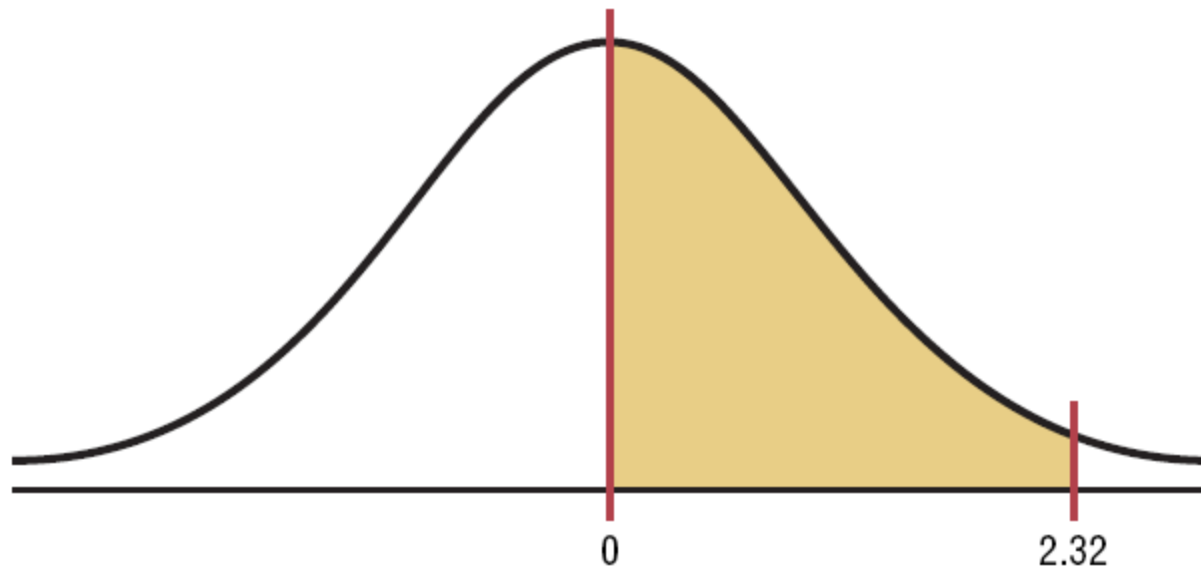
Find the area between $z = 1.68$ and $z = -1.37$.



The area is = .8682.

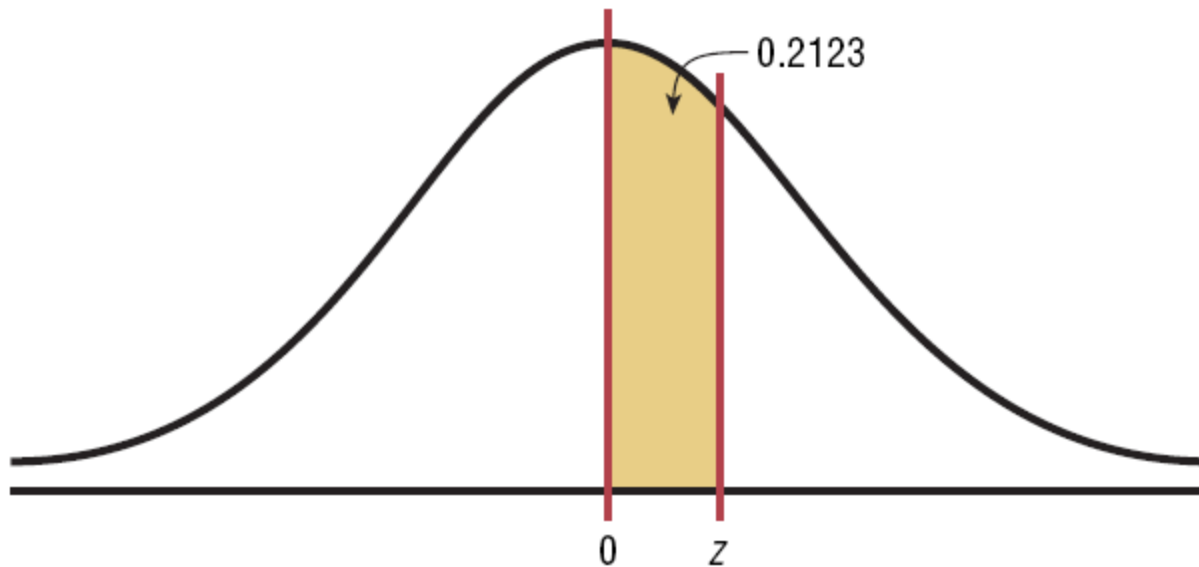
Probability

a. Find the probability: $P(0 < z < 2.32)$



Probability

Find the z value such that the area under the standard normal distribution curve between 0 and the z value is 0.2123.



Applications of the Normal Distributions

- The standard normal distribution curve can be used to solve a wide variety of practical problems. The only requirement is that the variable be normally or approximately normally distributed.

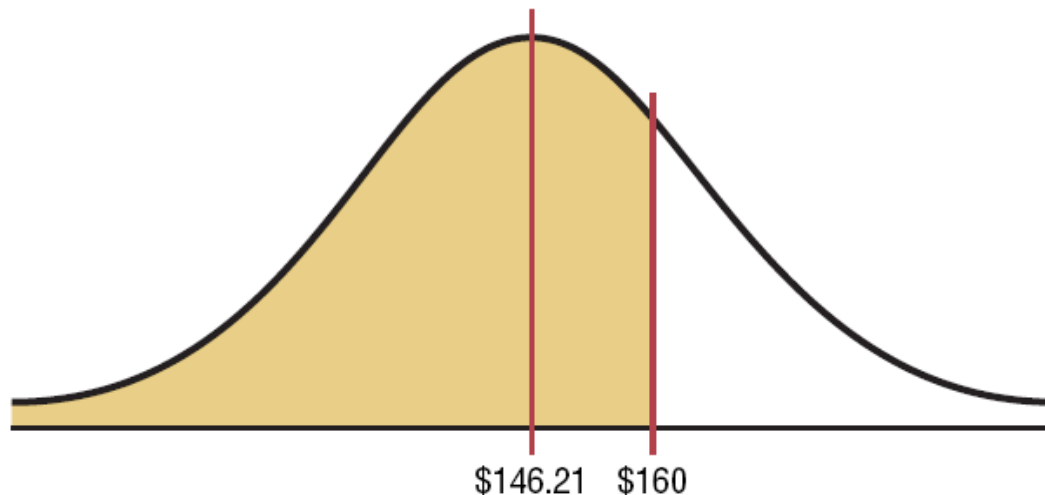
Applications of the Normal Distributions

- To solve problems by using the standard normal distribution, transform the original variable to a standard normal distribution variable by using the z value formula.
- This formula transforms the values of the variable into standard units or z values. Once the variable is transformed, the values from the standard normal distribution Table can be used to solve problems.

Holiday Spending

A survey by the National Retail Federation found that women spend on average \$146.21 for the Christmas holidays. Assume the standard deviation is \$29.44. Find the percentage of women who spend less than \$160.00. Assume the variable is normally distributed.

Step 1: Draw the normal distribution curve.



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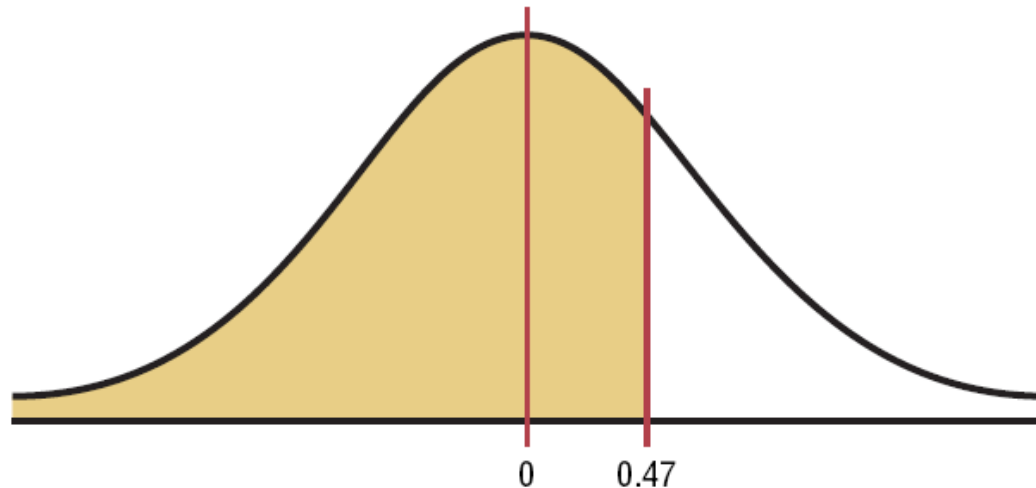
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Holiday Spending

Step 2: Find the z value corresponding to \$160.00.

$$z = \frac{X - \mu}{\sigma} = \frac{160.00 - 146.21}{29.44} = 0.47$$

Step 3: Find the area to the left of $z = 0.47$.



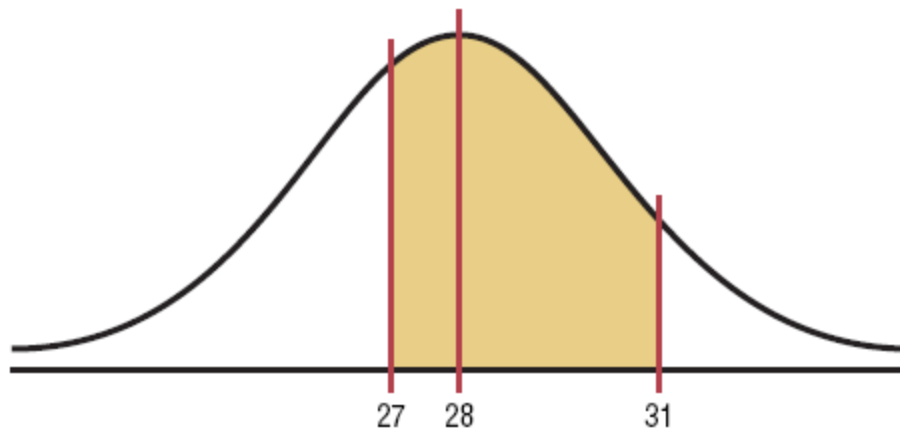
We obtain an area of .6808.

68% of women spend less than \$160.

Newspaper Recycling

Each month, an American household generates an average of 28 pounds of newspaper for garbage or recycling. Assume the standard deviation is 2 pounds. If a household is selected at random, find the probability of its generating between 27 and 31 pounds per month. Assume the variable is approximately normally distributed.

Step 1: Draw the normal distribution curve.



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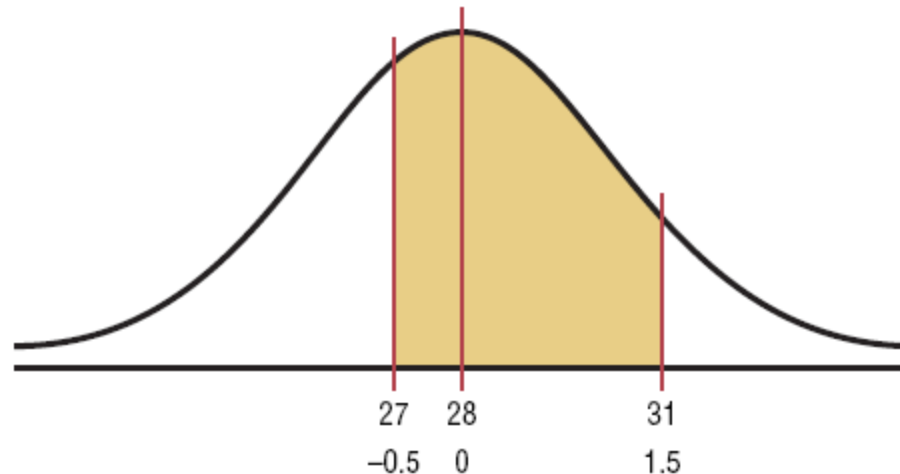
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Newspaper Recycling

Step 2: Find z values corresponding to 27 and 31.

$$z = \frac{27 - 28}{2} = -0.5 \qquad z = \frac{31 - 28}{2} = 1.5$$

Step 3: Find the area between $z = -0.5$ and $z = 1.5$.



We obtain an area of .6247.

The probability is 62%.

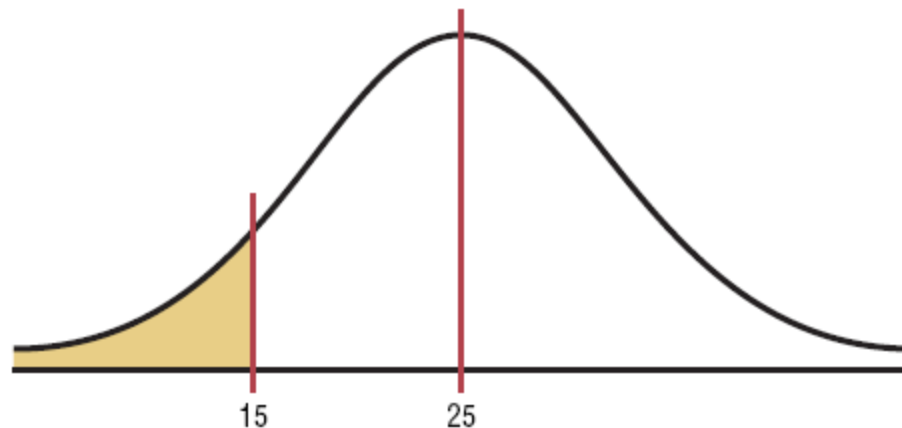
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Emergency Response

The American Automobile Association reports that the average time it takes to respond to an emergency call is 25 minutes. Assume the variable is approximately normally distributed and the standard deviation is 4.5 minutes. If 80 calls are randomly selected, approximately how many will be responded to in less than 15 minutes?

Step 1: Draw the normal distribution curve.



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Newspaper Recycling

Step 2: Find the z value for 15.

$$z = \frac{15 - 25}{4.5} = -2.22$$

Step 3: Find the area to the left of $z = -2.22$. It is 0.0132.

Step 4: To find how many calls will be made in less than 15 minutes, multiply the sample size 80 by 0.0132 to get 1.056. Hence, approximately 1 call will be responded to in under 15 minutes.



6.3 The Central Limit Theorem

In addition to knowing how individual data values vary about the mean for a population, statisticians are interested in knowing how the means of samples of the same size taken from the same population vary about the population mean.

Distribution of Sample Means

- A ***sampling distribution of sample means*** is a distribution obtained by using the means computed from random samples of a specific size taken from a population.
- ***Sampling error*** is the difference between the sample measure and the corresponding population measure due to the fact that the sample is not a perfect representation of the population.

Properties of the Distribution of Sample Means

- The mean of the sample means will be the same as the population mean.
- The standard deviation of the sample means will be smaller than the standard deviation of the population, and will be equal to the population standard deviation divided by the square root of the sample size.

The Central Limit Theorem

- As the sample size n increases, the shape of the distribution of the sample means taken with replacement from a population with mean μ and standard deviation σ will approach a normal distribution.
- The mean of the sample means equals the population mean. $\mu_{\bar{X}} = \mu$.
- The standard deviation of the sample means is called the **standard error of the mean**.

$$\sigma_{\bar{X}} = \sigma / \sqrt{n}.$$

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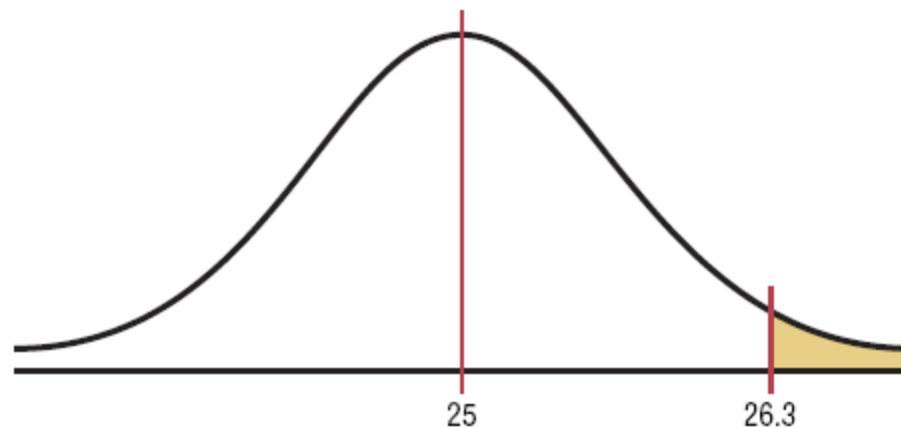
The Central Limit Theorem

- The central limit theorem can be used to answer questions about sample means in the same manner that the normal distribution can be used to answer questions about individual values.
- A new formula must be used for the z values:

$$z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Hours of Television

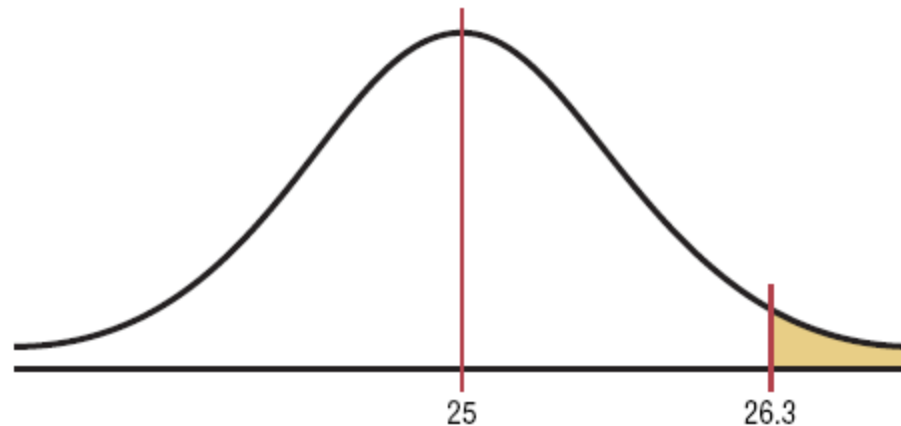
A. C. Nielsen reported that children between the ages of 2 and 5 watch an average of 25 hours of television per week. Assume the variable is normally distributed and the standard deviation is 3 hours. If 20 children between the ages of 2 and 5 are randomly selected, find the probability that the mean of the number of hours they watch television will be greater than 26.3 hours.



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Hours of Television



Since we are calculating probability for a sample mean, we need the Central Limit Theorem formula

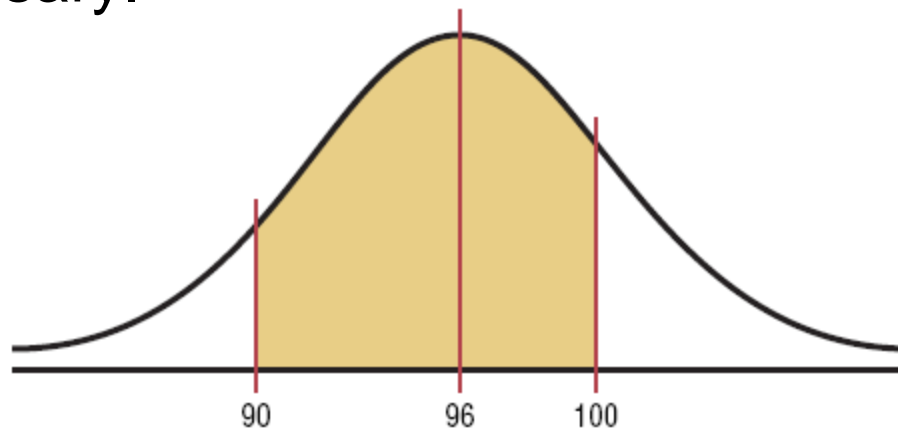
$$z = \frac{X - \mu}{\sigma / \sqrt{n}} = \frac{26.3 - 25}{3 / \sqrt{20}} = 1.94$$

The area is 0.0262. The probability of obtaining a sample mean larger than 26.3 hours is 2.62%.

Vehicle Age

The average age of a vehicle registered in the United States is 8 years, or 96 months. Assume the standard deviation is 16 months. If a random sample of 36 vehicles is selected, find the probability that the mean of their age is between 90 and 100 months.

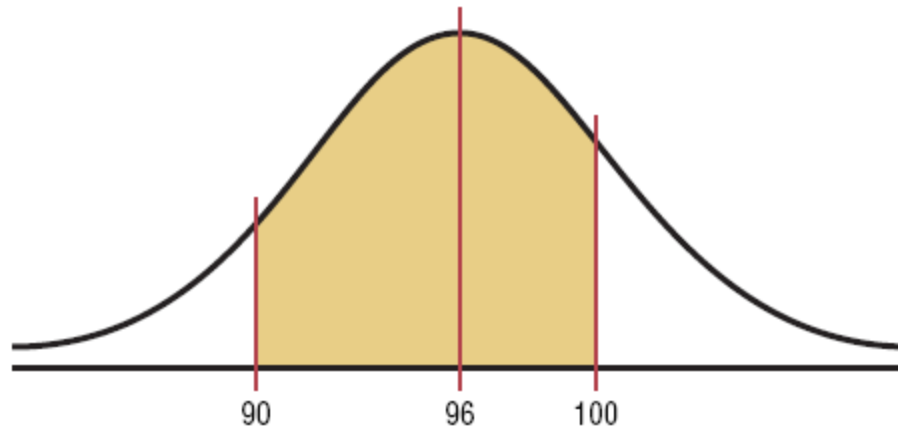
Since the sample is 30 or larger, the normality assumption is not necessary.



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Vehicle Age



$$z = \frac{90 - 96}{16/\sqrt{36}} = -2.25$$

$$z = \frac{100 - 96}{16/\sqrt{36}} = 1.50$$

Table E gives us areas 0.9332 and 0.0122, respectively.

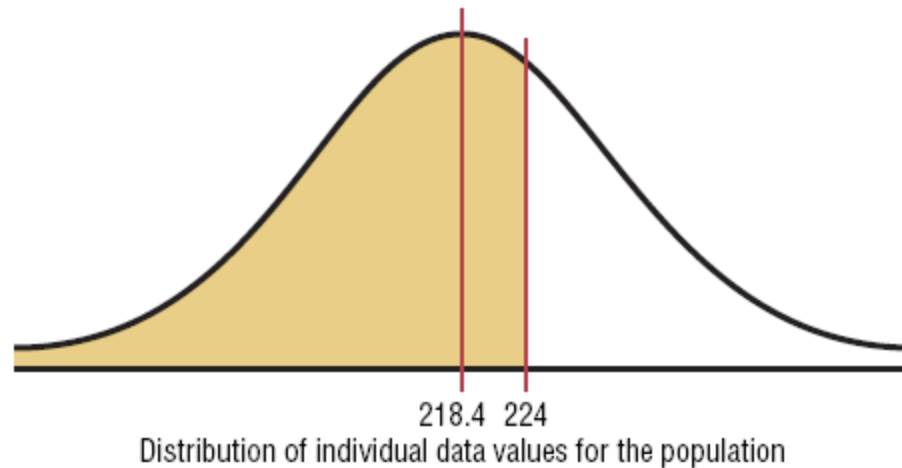
The desired area is $0.9332 - 0.0122 = 0.9210$.

The probability of obtaining a sample mean between 90 and 100 months is 92.1%.

Meat Consumption

The average number of pounds of meat that a person consumes per year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.

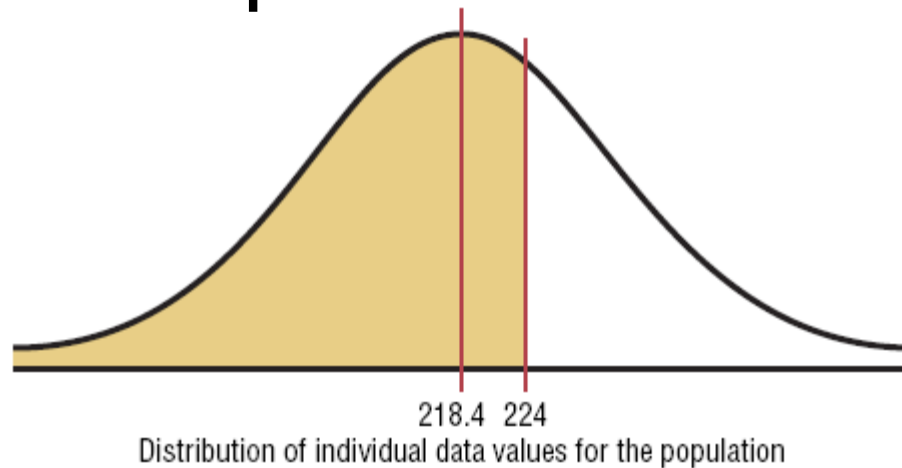
- a. Find the probability that a person selected at random consumes less than 224 pounds per year.



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Meat Consumption



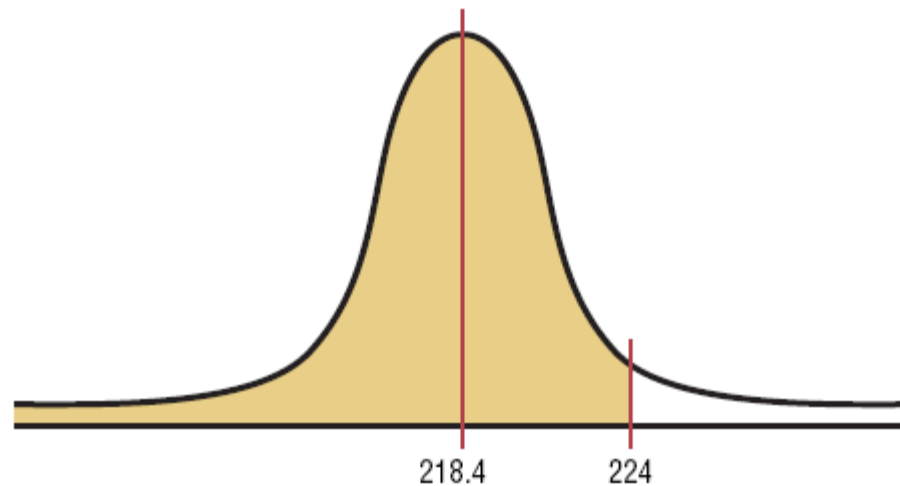
$$z = \frac{X - \mu}{\sigma} = \frac{224 - 218.4}{25} = 0.22$$

The area to the left of $z = 0.22$ is 0.5871. Hence, the probability of selecting an individual who consumes less than 224 pounds of meat per year is 0.5871, or 58.71%.

Meat Consumption

The average number of pounds of meat that a person consumes per year is 218.4 pounds. Assume that the standard deviation is 25 pounds and the distribution is approximately normal.

- b. If a sample of 40 individuals is selected, find the probability the sample will be less than 224 pounds per year.

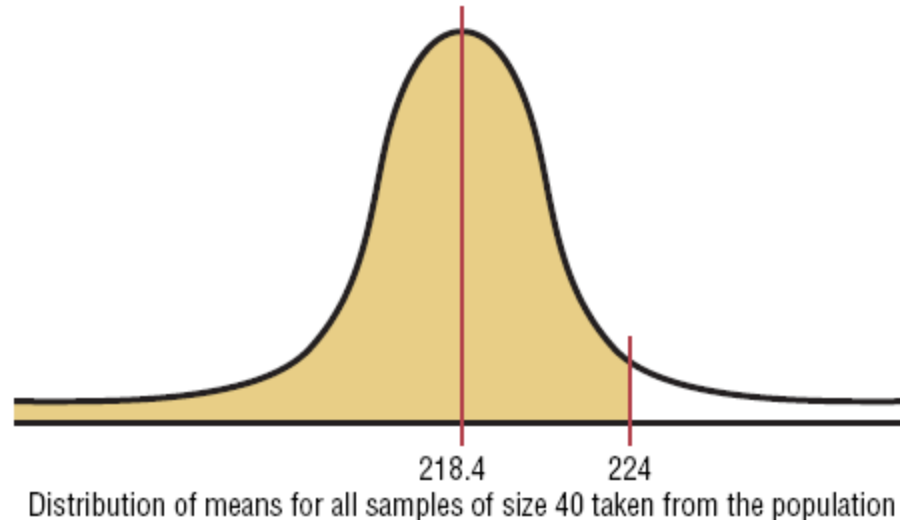


Distribution of means for all samples of size 40 taken from the population

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Meat Consumption



$$z = \frac{X - \mu}{\sigma / \sqrt{n}} = \frac{224 - 218.4}{25 / \sqrt{40}} = 1.42$$

The area to the left of $z = 1.42$ is 0.9222. Hence, the probability that the mean of a sample of 40 individuals is less than 224 pounds per year is 0.9222, or 92.22%.

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Finite Population Correction Factor

- The formula for standard error of the mean is accurate when the samples are drawn with replacement or are drawn without replacement from a very large or infinite population.
- A **correction factor** is necessary for computing the standard error of the mean for samples drawn without replacement from a finite population.

Finite Population Correction Factor

- The correction factor is computed using the following formula:

$$\sqrt{\frac{N - n}{N - 1}}$$

where N is the population size and n is the sample size.

- The standard error of the mean must be multiplied by the correction factor to adjust it for large samples taken from a small population.

Finite Population Correction Factor

$$\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}$$

- The standard error for the mean must be adjusted when it is included in the formula for calculating the z values.

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{N-1}}}$$